UNIVERSITY OF MUMBAI
SYLLABUS for the F.Y.B.A/B.Sc.
Programme: B.A./B.Sc.
Subject: Mathematics

Choice Based Credit System (CBCS)
with effect from the
academic year 2016-17
### Semester I

#### Calculus I

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#### Algebra I

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### Semester II

#### Calculus II

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#### Linear Algebra

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**Teaching Pattern**

1. Three lectures per week per course. Each lecture is of 1 hour duration.

2. One tutorial per week per course (the batches to be formed as prescribed by the University)
Syllabus for Semester I & II

**USMT101/UAMT101  CALCULUS I**

**Unit I: Real Number System** (15 Lectures)
Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, Absolute value $|.|$ and its properties.

AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.

Bounded sets, statement of l.u.b. axiom, g.l.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals.

**Unit II: Sequences** (15 Lectures)
Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

Convergence of standard sequences like

$$\left(\frac{1}{1+na}\right) \forall a > 0, \ (b^n) \forall 0 < b < 1, \ (c^\frac{1}{n}) \forall c > 0, \ & (n^\frac{1}{n})$$

algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences such as convergence of $(1 + \frac{1}{n})^n$.

Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequence, every convergent sequence is a Cauchy sequence and converse.

**Unit III: Limits & Continuity** (15 Lectures)
Brief review: Domain and range of a function, injective function, surjective function, bijective function, composite of two functions (when defined), Inverse of a bijective function.

Graphs of some standard functions such as $|x|$, $e^x$, $\log x$, $ax^2 + bx + c$, $\frac{1}{x}$, $x^n (n \geq 3)$, $\sin x$, $\cos x$, $\tan x$, $x \sin(\frac{1}{x})$, $x^2 \sin(\frac{1}{x})$ over suitable intervals of $\mathbb{R}$.
Definition of Limit \( \lim_{x \to a} f(x) \) of a function \( f(x) \), evaluation of limit of simple functions using the \( \epsilon - \delta \) definition, uniqueness of limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand-limit \( \lim_{x \to a^-} f(x) \), right-hand-limit \( \lim_{x \to a^+} f(x) \), non-existence of limits, \( \lim_{x \to \infty} f(x) \), \( \lim_{x \to -\infty} f(x) \) and \( \lim_{x \to a} f(x) = \pm \infty \).

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

Additional Reference Books

Tutorials for USMT101, UAMT101:
1) Application based examples of Archimedean property, intervals, neighbourhood. 2) Consequences of l.u.b. axiom, infimum and supremum of sets. 3) Calculating limits of sequences. 4) Cauchy sequences, monotone sequences. 5) Limit of a function and Sandwich theorem. 6) Continuous and discontinuous functions.

USMT102 ALGEBRA I

Prerequisites:
Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan’s laws, Cartesian product of two sets, Relations, Permutations \( nP_r \) and Combinations \( nC_r \).
Complex numbers: Addition and multiplication of complex numbers, modulus,
amplitude and conjugate of a complex number.

**Unit I: Integers & divisibility** (15 Lectures)
Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle.

Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers \( a \) \& \( b \) and that the g.c.d. can be expressed as \( ma + nb \) for some \( m, n \in \mathbb{Z} \), Euclidean algorithm, Primes, Euclid’s lemma, Fundamental theorem of arithmetic, The set of primes is infinite.

Congruences, definition and elementary properties, Euler’s \( \varphi \) function, Statements of Euler’s theorem, Fermat’s theorem and Wilson theorem, Applications.

**Unit II: Functions and Equivalence relations** (15 Lectures)
Definition of function; domain, co-domain and range of a function; composite functions, examples, Direct image \( f(A) \) and inverse image \( f^{-1}(B) \) for a function \( f \); Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion; Binary operation as a function, properties, examples.

Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.
Congruence is an equivalence relation on \( \mathbb{Z} \), Residue classes and partition of \( \mathbb{Z} \), Addition modulo \( n \), Multiplication modulo \( n \), examples.

**Unit III: Polynomials** (15 Lectures)
Definition of a polynomial, polynomials over the field \( F \) where \( F = \mathbb{Q}, \mathbb{R} \) or \( \mathbb{C} \), Algebra of polynomials, degree of polynomial, basic properties,

Division algorithm in \( F[X] \) (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem,

A polynomial of degree over \( n \) has at most \( n \) roots, Complex roots of a polyno-
mial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree in $\mathbb{C}[X]$ has exactly $n$ complex roots counted with multiplicity, A non constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number $\frac{p}{q}$ to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is a irrational number where $p$ is a prime number, roots of unity, sum of all the roots of unity.

Reference Books

Additional Reference Books
3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint 2013.

Tutorials:
1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).
SEMESTER II

USMT 201 CALCULUS II

Unit I: Series (15 Lectures)

Series $\sum_{n=1}^{\infty} a_n$ of real numbers, simple examples of series, Sequence of partial sums of a series, convergence of a series, convergent series, divergent series, Necessary condition: $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow a_n \rightarrow 0$, but converse is not true, algebra of convergent series, Cauchy criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ ($p > 1$), Comparison test, limit comparison test, alternating series, Leibnitz’s theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), root test (without proof), and examples.

Unit II: Limits & Continuity of functions (15 Lectures)

Definition of Limit $\lim_{x \to a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\epsilon - \delta$ definition, uniqueness of limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand-limit $\lim_{x \to a^-} f(x)$, right-hand-limit $\lim_{x \to a^+} f(x)$, non-existence of limits, $\lim_{x \to -\infty} f(x)$, $\lim_{x \to \infty} f(x)$ and $\lim_{x \to a} f(x) = \infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity. Intermediate value theorem and its applications, Bolzano- Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

Differentiation of real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely,
algebra of differentiable functions.

Chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples).

**Unit III: Applications of differentiation** (15 Lectures)
Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, points of inflection.

Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing and decreasing function, examples,

L-hospital rule without proof, examples of indeterminate forms, Taylor’s theorem with Lagrange’s form of remainder with proof, Taylor polynomial and applications.

**Reference Books:**

**Additional Reference Books:**

**Tutorials:** 1. Calculating limit of series, Convergence tests. 2. Properties of continuous functions. 3. Differentiability, Higher order derivatives, Leibnitz theorem. 4. Mean value theorems and its applications. 5. Extreme values, increasing and decreasing functions. 6. Applications of Taylors theorem and Taylors polynomials.

**USMT 202/ UAMT 201 LINEAR ALGEBRA**

**Prerequisites:** Review of vectors in \( \mathbb{R}^2, \mathbb{R}^3 \) and as points, Addition and scalar multiplication of vectors in terms of co-ordinates, dot-product structure, Scalar
triple product, Length (norm) of a vector.

Unit I: System of Linear equations and Matrices (15 Lectures)
Parametric equation of lines and planes, system of homogeneous and non-
homogeneous linear equations, the solution of system of $m$ homogeneous linear
equations in $n$ unknowns by elimination and their geometrical interpretation for
$(n, m) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$; definition of $n$–tuples of real
numbers, sum of two $n$–tuples and scalar multiple of an $n$–tuple.

Matrices with real entries; addition, scalar multiplication and multiplication of
matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix,
scalar matrices, diagonal matrices, upper triangular matrices, lower triangu-
lar matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices;
identities such as $(AB)^t = B^t A^t, (AB)^{-1} = B^{-1} A^{-1}$.

System of linear equations in matrix form, elementary row operations, row
echelon matrix, Gaussian elimination method, to deduce that the system of $m$
homogeneous linear equations in $n$ unknowns has a non-trivial solution if $m < n$.

Unit II: Vector spaces (15 Lectures)
Definition of a real vector space, examples such as $\mathbb{R}^n$, $\mathbb{R}[X]$, $M_{m \times n}(\mathbb{R})$, space
of all real valued functions on a non empty set.

Subspace: definition, examples: lines, planes passing through origin as sub-
spaces of $\mathbb{R}^2, \mathbb{R}^3$ respectively; upper triangular matrices, diagonal matrices, sym-
metric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$ ($n = 2, 3$);
$P_n(X) = \{a_0 + a_1 X + \cdots + a_n X^n | a_i \in \mathbb{R} \forall 0 \leq i \leq n\}$ as a subspace $\mathbb{R}[X]$, the space of all solutions of the system of $m$
homogeneous linear equations in $n$ unknowns as a subspace of $\mathbb{R}^n$.

Properties of a subspace such as necessary and sufficient condition for a non
empty subset to be a subspace of a vector space, arbitrary intersection of sub-
spaces of a vector space is a subspace, union of two subspaces is a subspace if
and only if one is a subset of the other.

Finite linear combinations of vectors in a vector space; the linear span $L(S)$
of a non-empty subset $S$ of a vector space, $S$ is a generating set for $L(S)$, $L(S)$
is a vector subspace of $V$; linearly independent/linearly dependent subsets of a
vector space, a subset $\{v_1, v_2, \cdots, v_k\}$ of a vector space is linearly dependent if
and only if $\exists i \in \{1, 2, \cdots k\}$ such that $v_i$ is a linear combination of the other
Unit III: Basis and Linear Transformations (15 Lectures)

Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two basis of a vector space have the same number of elements, any set of $n$ linearly independent vectors in an $n$-dimensional vector space is a basis, any collection of $n+1$ linearly independent vectors in an $n$-dimensional vector space is linearly dependent; if $W_1, W_2$ are two subspaces of a vector space $V$ then $W_1 + W_2$ is a subspace of the vector space $V$ of dimension $\dim(W_1) + \dim(W_1) - \dim(W_1 \cap W_2)$, extending any basis of a subspace $W$ of a vector space $V$ to a basis of the vector space $V$.

Linear transformations; kernel $\ker(T)$ of a linear transformation, matrix associated with a linear transformation, properties such as: for a linear transformation $T$ kernel($T$) is a subspace of the domain space of $T$ and the image $\text{image}(T)$ is a subspace of the co-domain space of $T$. If $V,W$ are real vector spaces with $\{v_1, v_2, \cdots, v_n\}$ a basis of $V$ and $\{w_1, w_2, \cdots, w_n\}$ any vectors in $W$ then there exists a unique linear transformation $T : V \rightarrow W$ such that $T(v_j) = w_j \forall 1 \leq j \leq n$, Rank nullity theorem (statement only) and examples.

Reference Books:

Additional Reference Books:
10. Gareth Williams: Linear Algebra with Applications.
Tutorials:
1) Solving homogeneous system of m equations in n unknowns by elimination for \((m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\), row echelon form.
2) Solving system \(Ax = b\) by Gauss elimination, Solutions of system of linear Equations.
3) Verifying whether given \((V, +, \cdot)\) is a vector space with respect to addition + and scalar multiplication \(\cdot\).
4) Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5) Finding basis of a vector space such as \(P_3(X), M_3(\mathbb{R})\) etc. Verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.
6) Verifying whether a map \(T : X \rightarrow Y\) is a linear transformation, finding kernel of a linear transformation and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

Scheme of Examination

There will be a Semester end external Theory examination of 100 marks for all the courses of Semester I & II.
1. Duration: The examinations shall be of 3 Hours duration.
2. Question Paper Pattern: There shall be FOUR questions. The first three questions shall be of 25 marks on each unit, and the fourth question shall be of 25 marks based on Unit I, II, & III.
3. All the questions shall be compulsory with internal choices within the questions. Including the choices, the marks for each question shall be 38-40.
4. Questions may be subdivided into sub questions as a, b, c, d & e, etc & the allocation of marks depends on the weightage of the topic.